Topic 4: Surds

Any number which **can** be written as a fraction of two integers (whole numbers) is known as a **rational** number, e.g. $\frac{3}{2}$, 4.5, 7, $\sqrt{36}$, $1\frac{2}{3}$ are all rational.

Any number which **cannot** be written as a fraction of two integers is known as an **irrational** number. The most famous one is π . Square roots of numbers (which are not square numbers) are also irrational e.g. $\sqrt{37}, \sqrt{28}$ etc. Irrational numbers in this form are called **surds**.

Rules of Surds

$$\sqrt{a} \times \sqrt{b} = \sqrt{ab}$$

$$\sqrt{a} \div \sqrt{b} = \sqrt{\frac{a}{b}}$$

$$a\sqrt{c} \pm b\sqrt{c} = (a \pm b)\sqrt{c}$$

Example: Without using a calculator, simplify

1. $\sqrt{50} \div \sqrt{2}$

$= \sqrt{\frac{50}{2}} = \sqrt{25} = 5$

2. $(1 + 4\sqrt{2})(3 - \sqrt{2})$

$= 3 - \sqrt{2} + 12\sqrt{2} - 4(2)$

$= 3 - \sqrt{2} + 12\sqrt{2} - 8$

$= -5 + 11\sqrt{2}$

3. (a.) $\sqrt{72}$ (b.) $\sqrt{800}$

(a.)

* try to look for a square factor

$\sqrt{72} = \sqrt{36} \times \sqrt{2}$

$= 6\sqrt{2}$

(b.)

$\sqrt{800} = \sqrt{100} \times \sqrt{8}$

$= 10\sqrt{8}$ can be simplified again

$= 10 \times \sqrt{4} \times \sqrt{2}$

$= 10 \times 2 \times \sqrt{2} = 20\sqrt{2}$

Rationalising the denominator

This is the process of eliminating surds from the denominator.

For expressions of the type $\frac{1}{\sqrt{a}}$ multiply top and bottom by $\sqrt{a}$.

For expressions of the type $\frac{1}{b+\sqrt{a}}$ multiply top and bottom by $b-\sqrt{a}$.

Example: Rationalise the following

1. $\frac{2}{\sqrt{5}}$

$$\frac{2}{\sqrt{5}} = \frac{2}{\sqrt{5}} \times \frac{\sqrt{5}}{\sqrt{5}}$$
$$= \frac{2\sqrt{5}}{5} \checkmark$$

2. $\frac{8}{\sqrt{3}-2}$

$$\frac{8}{\sqrt{3}-2} = \frac{8}{\sqrt{3}-2} \times \frac{\sqrt{3}+2}{\sqrt{3}+2}$$
$$= \frac{8(\sqrt{3}+2)}{(3)+2\sqrt{3}-2\sqrt{3}-4}$$
$$= \frac{8(\sqrt{3}+2)}{-1}$$
$$= -8(\sqrt{3}+2) \checkmark$$

Example: Given that $\sqrt{2} \approx 1.414$ calculate $\frac{1}{1+\sqrt{2}}$

$$\frac{1}{1+\sqrt{2}} = \frac{1}{1+\sqrt{2}} \times \frac{1-\sqrt{2}}{1-\sqrt{2}}$$
$$= \frac{1-\sqrt{2}}{1-\sqrt{2}+\sqrt{2}-(2)}$$
$$= \frac{1-\sqrt{2}}{-1}$$
$$= \sqrt{2}-1$$
$$= 1.414-1$$
$$= 0.414$$